Institut de Physique Théorique
CEA-Direction de la Recherche Fondamentale
Unité mixte de recherche CNRS 3681

Cours de physique théorique

agréé par *l'École doctorale « Physique en Île-de-France » – ED PIF*

Quantum integrability: old and new tools

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In recent years, the spectral problem of planar *N=4 Super Yang-Mills* quantum field theory (N=4 SYM) was understood with a great precision, giving the opportunity to check an example of the *Anti de Sitter/Conformal Field Theory* (AdS/CFT) correspondence in unexpected detail. The spectacular advancements in this four-dimensional interacting gauge theory were enabled by the presence of hidden integrable structures. The progress was achieved through a decade-long effort of the scientific community that had combined old ideas from integrable *spin chains* and two-dimensional *sigma models* with new ones. In this course we aim to show how the new and old ideas work together producing a solution of the planar N=4 SYM spectral problem in terms of the *quantum spectral curve* (QSC).

The course is divided into two parts. The first part is an introduction to some standard integrability techniques and how they appear in the context of the AdS/CFT correspondence. The first two lectures will be devoted to the treatment of spin chains, occurring at weak coupling, and sigma models, occurring at strong coupling. The first lecture introduces the prototypical example of spin chains, the *XXX model*, and its solution in terms of coordinate and algebraic *Bethe ansatz*. The second lecture is devoted to the finite gap solution of the *principal chiral model* as an example of integrable string sigma model. In the third lecture the full problem of the spectrum of the N=4 SYM theory will be set up. This will be done by detailing the N=4 superconformal algebra *psu(2,2|4)* and its representations in term of oscillators.

In the fourth lecture we establish a bridge to newly developed techniques by revising rational spin chains once more, now focusing on *Hirota equations* and *QQ- and TQ-relations*. We will review the remarkably simple Wronskian-type solution and their underlying geometric structure.

In the fifth lecture we introduce the QSC and show that it can be understood as a monodromy requirement, or Riemann-Hilbert-type constraint, on the above-mentioned Wronskian construction. The coupling constant of the gauge theory appears only as a parameter determining the position of branch points of the Riemann surface.

In the last lecture we review the most recent techniques developed to solve QSC explicitly, including a very efficient perturbative analytic algorithm that allows to reach very high loop orders in perturbation theory.

1) XXX spin chain, algebraic/coordinate Bethe ansatz, T-Q relations.
2) Quasiclassical limit: sigma models, Lax connection, finite gap solution.
3) Oscillator representation of psu(2,2|4) and the N=4 SYM spectrum.
4) Q-system for rational spin chains. Hirota equation on T-hooks and geometry behind its Wronskian solution.
5) AdS/CFT quantum spectral curve (QSC) as a Q-system and as a Riemann-Hilbert problem.
6) Explicit solution of QSC in different regimes, perturbative analytic computations, multiple zeta values, and the 10-loop Konishi anomalous dimension.

Lieu *: IPhT, CEA Saclay, Orme des Merisiers, Bât. 774, p. 1A Salle C. Itzykson.*
Accès *: navettes CEA du RER B Le Guichet vers CEA Ormes, toutes les 15 minutes de 8h00 à 9h45, ou bus publics Mobicaps 9 et 10, Albatrans 91.06 et 91.10.*
Renseignements *:* ***http://ipht.cea.fr*** *ou* ***ipht-lectures@cea.fr***

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