Cours de Physique Théorique de l'IPhT, année 2009-2010

Organisés en collaboration avec

Ecole Doctorale de Physique de la Région Parisienne - ED 107

Chaotic dynamical systems

Stéphane
Nonnenmacher

Institut de Physique Théorique

Les vendredis 18/9, 25/9, 2/10, 9/10, 16/10, 23/10/2009.

The aim of this course is to present some properties of low-dimensional dynamical systems, particularly in the case where the dynamics is "chaotic". We will describe several aspects of "chaos", by introducing various "modern" mathematical tools, allowing us to analyze the long-time properties of such systems. Several simple examples, leading to explicit computations, will be treated in detail. A tentative plan (not necessarily chronological) follows.

- Definition of a dynamical system: flow generated by a vector field, discrete time transformation. Poincaré sections vs. suspended flows. Examples: Hamiltonian flow, geodesic flow, transformations on the interval or on the two-dimensional torus.
- Ergodic theory: long-time behavior. Statistics of long periodic orbits. Probability distributions invariant through the dynamics (invariant measures). "Physical" invariant measure.
- Chaotic dynamics: instability (Lyapunov exponents) and recurrence. From the hyperbolic fixed point to Smale's horseshoe.
- Various levels of chaos: ergodicity, weak and strong mixing.
- Symbolic dynamics: subshifts on 1D spin chains. Relation (semiconjugacy) with expanding maps on the interval.
- Uniformly hyperbolic systems: stable/unstable manifolds. Markov partitions: relation with symbolic dynamics. Anosov systems. Example: Arnold's "cat map" on the two-dimensional torus.
- Complexity theory. Topological entropy, link with statistcs of periodic orbits. Partitions functions (dynamical zeta functions). Kolmogorov-Sinai entropy of an invariant measure.
- Exponential mixing of expanding maps: spectral analysis of some transfer operator. Perron-Frobenius theorem.
- Structural stability vs. bifurcations. Examples: logistic map on the interval/non-linear perturbation of the "cat map".

Horaires : les vendredis de 10h00 à 12h15.

Lieu : IPhT, CEA Saclay, Orme des Merisiers, Bât. 774, p.1A Salle C. Itzykson.

Accès : Par lignes de bus publics (269.02 et 91.06) ou
- navettes CEA: RER B Le Guichet vers CEA Orme Bât. 774, toutes les 15 min de 8h00 à 9h45;
- navette CEA: CEA Orme Bât. 774 vers RER B Le Guichet à 12h36.

Renseignements : `http://ipht.cea.fr` ou `ipht-lectures@cea.fr`

(Mise à jour affiche: 2009-07-15)